Combinatorics

Semestral Examination

Instructions: All questions carry equal marks.

1. Prove that a $k \times n$ Latin rectangle with $k < n$ can be completed to a $n \times n$ Latin square.

2. Prove that for any natural number n, the number of mutually orthogonal Latin squares of order n is at most $n - 1$. Show that this bound is attained in case n is a power of a prime number.

3. Define an hyperoval in a projective plane of order n. Prove that in the projective plane of order 4, any three non collinear points are contained in exactly three distinct hyperovals.

4. Define primitive group actions. Prove that any primitive group action is transitive, but the converse need not be true. Prove that any 2-transitive action is primitive.

5. Let G be a connected regular graph of degree k with adjacency matrix A. Prove that the vector whose all coordinates equal 1 is an eigen vector of A with eigen value k. Further, prove that this eigen value k has multiplicity 1 for A.